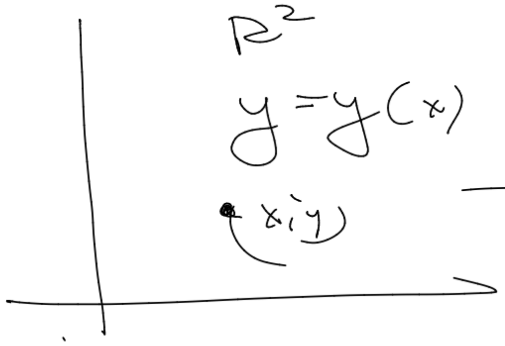
R^2

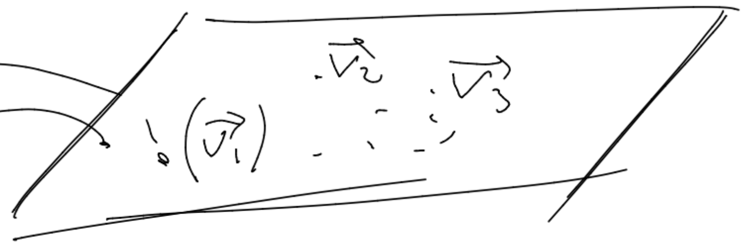
$y = y(x)$

(x, y)



R^5

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$



$\boxed{\text{Линейные}}$

$v_1 + v_2 \in L$

$\alpha \cdot v_1 \in L$

$\begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & v_4 \end{pmatrix} \Rightarrow A = \begin{pmatrix} \cdots \end{pmatrix}$

$A = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

R^5

$\boxed{\{ \text{---} \}}$ СЛАУ

лин

$\boxed{A\vec{x} = \vec{b}}$

dataset

{векторн. пр-ва}

– базис

↳ # пер-х

↳ PCA

↳ dim ↓

$\boxed{\text{лин-ть}}$

мн-во и простр-ва

вектр-ва ⟺ лин. + 10

v_1 $\boxed{v_1 + v_2}$

v_2 $\boxed{\alpha v_1}$

мн-во ≠ лин. пр-ва

3

1

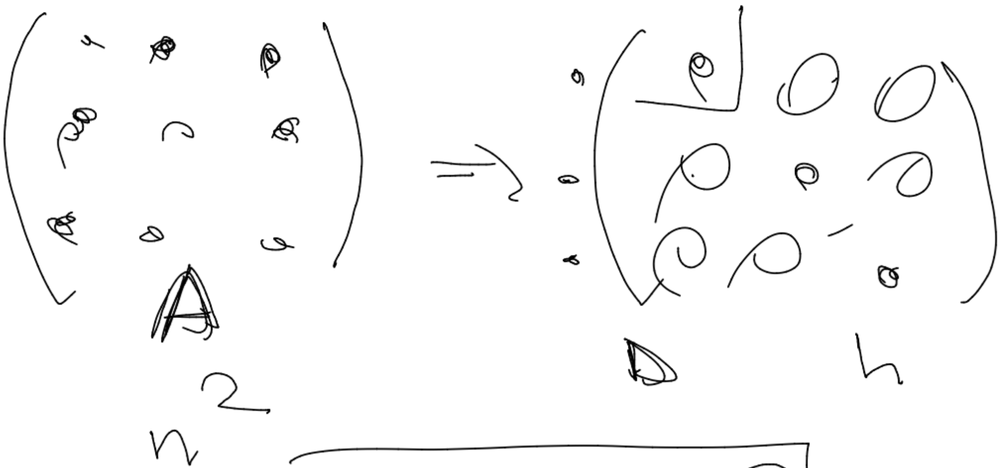
5

$\mathbb{R}$ +

$$A \cdot \vec{x} = \vec{b}$$

$A_{n\times n}$

1) методу $\boxed{n^3}$



$\boxed{ст} \rightarrow \boxed{}$ нов

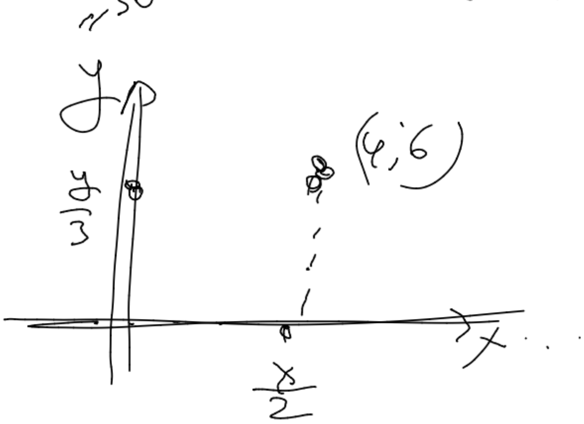
$$\boxed{A = P \cdot D \cdot P^{-1}}$$

ст.

ст → нов

$P^{-1} \exists \Leftrightarrow \det P \neq 0$

$P_{n\times n}$



$$A = \begin{pmatrix} 7 & 1 \\ 3 & 5 \end{pmatrix}$$

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$$\boxed{A \cdot \vec{v} = \lambda \cdot \vec{v}} \; !$$

собств. вектора ?

$A\vec{v} - \lambda\vec{v} = \vec{0}$

$(*)$ $(A - \lambda \cdot I)\vec{v} = \vec{0}$ $(*)$

$$\det \begin{pmatrix} 7-\lambda & 1 \\ 3 & 5-\lambda \end{pmatrix} = (7-\lambda)(5-\lambda) - 3 = 0$$

$$= 35 - 12\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 12\lambda + 32 = 0$$

собств-е числа $\rightarrow$ $\begin{cases} \lambda_1 = 4 \\ \lambda_2 = 8 \end{cases}$ $\leftarrow$ eigenvalue

$\lambda_1 = 4$ $v_1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$3a + b = 0$

$\lambda_2 = 8$ $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \vec{0} \Rightarrow -1a + b = 0$$

$$A = \begin{pmatrix} 7 & 1 \\ 3 & 5 \end{pmatrix}_{2\times 2} \rightarrow \begin{array}{l} \lambda_1 = 4 \rightarrow v_1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \\ \lambda_2 = 8 \rightarrow v_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{array}$$

$$\boxed{A = P \cdot D \cdot P^{-1}}$$

$$D = \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix}$$

$$P = \begin{pmatrix} v_1 & v_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}$$ cr → wb

1

$$A^3 = A \cdot A \cdot A = P D \cancel{P^{-1} \cdot P} D \cancel{P^{-1} \cdot P} D P^{-1} = P \cdot D \cdot D D \cdot P^{-1}$$

$$= P \cdot D^3 \cdot P^{-1} = P \cdot \begin{bmatrix} 4^3 & 0 \\ 0 & 8^3 \end{bmatrix} \cdot P^{-1}$$

$$f(A) = P\, f(D) \cdot P^{-1}, \quad f(D) = \begin{bmatrix} f(\lambda_1) & 0 & 0 \\ 0 & f(\lambda_2) & 0 \\ 0 & 0 & f(\lambda_3) \end{bmatrix}$$

II,

$$A=\begin{pmatrix}7 & 1\\ 3 & 5\end{pmatrix} \qquad \lambda_1=4 \quad v_1=\begin{pmatrix}-1\\3\end{pmatrix} \qquad \lambda_2=8 \quad v_2=\begin{pmatrix}2\\2\end{pmatrix} \qquad Ax=\vec{b}$$

$\boxed{x_t}$, $\boxed{y_t}$ $\quad x_t=x(t)$, $y_t=y(t)$

$$\Rightarrow \begin{cases} x_{t+1}=7x_t+y_t \\ y_{t+1}=3x_t+5y_t \end{cases} \Rightarrow \begin{pmatrix}x_{t+1}\\y_{t+1}\end{pmatrix}=\begin{bmatrix}7 & 1\\ 3 & 5\end{bmatrix}\begin{pmatrix}x_t\\y_t\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x_{t+1}\\y_{t+1}\end{pmatrix}=A\cdot\begin{pmatrix}x_t\\y_t\end{pmatrix} \Rightarrow$$

$$10\cdot 1=10$$

$$\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{matrix}1 & 2\\ 3 & 4\end{matrix}$$

$$\underset{2\times1}{\begin{pmatrix}x_{t+1}\\y_{t+1}\end{pmatrix}}=\underset{2\times2}{P}\,\underset{2\times2}{D}\,\underset{2\times2}{P^{-1}}\underset{2\times1}{\begin{pmatrix}x_t\\y_t\end{pmatrix}} \Rightarrow P^{-1}\begin{pmatrix}x_{t+1}\\y_{t+1}\end{pmatrix}=D\,P^{-1}\begin{pmatrix}x_t\\y_t\end{pmatrix}$$

$=$ Замена:

$$\underset{(2\times2)}{P^{-1}}\cdot\underset{(2\times1)}{\begin{pmatrix}x_t\\y_t\end{pmatrix}}=\begin{pmatrix}\tilde{x}_t\\ \tilde{y}_t\end{pmatrix}, \quad P^{-1}\begin{pmatrix}x_{t+1}\\y_{t+1}\end{pmatrix}=\begin{pmatrix}\tilde{x}_{t+1}\\ \tilde{y}_{t+1}\end{pmatrix}$$

$$\begin{pmatrix}\tilde{x}_{t+1}\\ \tilde{y}_{t+1}\end{pmatrix}=\begin{pmatrix}4 & 0\\ 0 & 8\end{pmatrix}\begin{pmatrix}\tilde{x}_t\\ \tilde{y}_t\end{pmatrix} \Rightarrow \begin{cases}\tilde{x}_{t+1}=4\tilde{x}_t \Rightarrow \tilde{x}_t=C_1\cdot 4^t \\ \tilde{y}_{t+1}=8\tilde{y}_t \Rightarrow \tilde{y}_t=C_2\cdot 8^t\end{cases}$$

$$\begin{cases} \tilde{x}_t = c_1 \cdot 4^t \\ \tilde{y}_t = c_2 \cdot 8^t \end{cases} \qquad P^{-1}\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} \tilde{x}_t \\ \tilde{y}_t \end{pmatrix}$$

$$P^{-1} \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} c_1 \cdot 4^t \\ c_2 \cdot 8^t \end{pmatrix}$$

$$\cancel{P P^{-1}} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = P \begin{pmatrix} c_1 4^t \\ c_2 \cdot 8^t \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} c_1 4^t \\ c_2 8^t \end{pmatrix} = \begin{pmatrix} -c_1 4^t + 2c_2 8^t \\ 3c_1 4^t + 2c_2 8^t \end{pmatrix}$$

<u>Otber</u>: $$\begin{cases} x_t = -c_1 4^t + 2c_2 8^t \\ y_t = 3c_1 4^t + 2c_2 8^t \end{cases}$$